



Tolling, Collusion and Equilibrium Problems with Equilibrium Constraints

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Abstract

An Equilibrium Problem with an Equilibrium Constraint (EPEC) is a mathematical construct that can be applied to private competition in highway networks. In this paper we consider the problem of finding a Nash Equilibrium in a situation of competition in toll pricing on a network utilising two alternative algorithms. In the first algorithm, we utilise a Gauss Seidel fixed point approach based on the cutting constraint algorithm for toll pricing. The second algorithm that we propose, a novel contribution of this paper, is the extension of an existing sequential linear complementarity programming approach for finding the competitive Nash equilibrium when there is a lower level equilibrium constraint. Finally we develop an intuitive approach to represent collusion between players and demonstrate that as the level of collusion goes from none to full collusion so the solution maps from the Nash to monopolistic solution. However we also show that there may be local solutions for the collusive monopoly which lie closer to the second best welfare toll solution than does the competitive Nash equilibrium.

Keywords: Sequential Linear Complementarity Programming (SLCP); EPEC; Competition; Nash Equilibrium; Collusion.

1. Introduction

The motivation of the research in this paper stems from the observation that in recent years there has been increasing amount of private sector participation within areas that are conventionally the privy of the public purse. The driving force behind this change is the assumed higher efficiency of the private sector coupled with increasing public pressures on governments for accountability and the corresponding need to derive value for money from their various budgetary commitments which are ultimately funded by the tax paying public.

In highway transportation, privately operated roads are not novel concepts (Viton, 1995). However there has been little analysis on this topic in terms of the competition between private sector providers and the equilibrium outcomes, save for theoretical

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